

CP Violation in B Decays

Hitoshi Yamamoto

The University of Hawaii

2505 Correa Rd, Honolulu, HI 96822, USA

E-mail: hitoshi@phys.hawaii.edu

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Abstract

We review the physics of CP violation in B decays. After introducing the CKM matrix and how it causes CP violation, we cover three types of CP violation that can occur in B decays: CP violation in mixing, CP violation by mixing-decay interference, and CP violation in decay.

1 CP Violation and the CKM Matrix

1.1 CP Transformation of the quark- W interaction

General left-handed quark- W interaction can be written in the interaction picture as (for 3 generations of quarks)

$$L_{\text{int}}(t) = \int d^3x \left(\mathcal{L}_{qW}(x) + \mathcal{L}_{qW}^\dagger(x) \right) \quad (1)$$

which is the space-integral of the Lagrangian density given by

$$\mathcal{L}_{qW}(x) = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} V_{ij} \bar{U}_i(x) \gamma_\mu (1 - \gamma_5) D_j(x) W^\mu(x) \quad (2)$$

where U_i and D_i are the up-type and down-type quark fields

$$U_i(x) \equiv \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad D_j(x) \equiv \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix} \quad (x \equiv (t, \vec{x})) \quad (3)$$

and the coupling of the $V - A$ quark currents to W is given by the complex parameters V_{ij} forming a 3×3 matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (4)$$

called the Cabbibo-Kobayashi-Masukawa (CKM) matrix.

The CP transformation exchanges particle n and its antiparticle \bar{n} , flips the momentum, and keeps the spin z component σ unchanged. In terms of creation operators,

$$(CP)a_{n,\vec{p},\sigma}^\dagger(CP)^\dagger \equiv \eta_n a_{\bar{n},-\vec{p},\sigma}^\dagger \quad (5)$$

where η_n is an arbitrary phase factor that in general can depend on particle type (except that those of a particle and its antiparticle are related by $\eta_{\bar{n}} = (-)^{2J}\eta_n^*$ where J is the spin of the particle) that in essence defines the CP operator in the Hilbert space. Any choice of the CP phases gives a legitimate CP operator. For some choices, however, a given interaction Lagrangian may commute with the CP operator, and if such choice can be made, then the processes caused by the interaction is invariant under CP .

Quark and W fields are made of creation and annihilation operators, and thus the transformation property (5) leads to those of fields. Then, a straightforward algebra shows that the quark- W interaction (2) transforms as (setting the irrelevant phase of W , η_W , to be unity)

$$(CP)\mathcal{L}_{qW}(CP)^\dagger = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} \eta_{U_i}\eta_{D_j}^* V_{ij} (\bar{U}_i\gamma^\mu(1-\gamma_5)D_jW_\mu)^\dagger, \quad (6)$$

where the space-time argument x on the left changed to $x' = (t, -\vec{x})$ on the right which has no significance when integrated over space. Then, if one can choose the phases such that

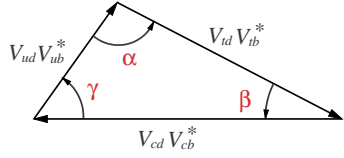
$$\eta_{U_i}\eta_{D_j}^* V_{ij} = V_{ij}^*, \quad (7)$$

then we have $(CP)\mathcal{L}_{qW}(CP)^\dagger = \mathcal{L}_{qW}^\dagger$ and the two terms in (1) simply swaps keeing the interaction Lagrangian invariant under CP . Given that η_{U_i} and η_{D_j} are arbitrary phases associated with each quark, the condition above is equivalent to being able to rotate quark phases to make all V_{ij} real without changing \mathcal{L}_{qW} . We can always make 5 of V_{ij} real since there are 6 quarks which have 5 relative phases.

1.2 Unitarity triangle

So far, we dealt with a completely general 3×3 matrix V . In the standard model, the CKM matrix is written as $V = S^{u\dagger}S^d$ where $S^{u(d)}$ is the unitary matrix that transforms the left-handed part of u -type (d -type) quarks in the bi-unitary diagonalization of the mass matrices; namely, V is unitary. Then, the orthogonality relation of the d -column and b -column can be written as a

triangle relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (8)$$


where the angles are defined by ¹

$$\alpha \equiv \arg \left(\frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*} \right), \quad \beta \equiv \arg \left(\frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*} \right), \quad \gamma \equiv \arg \left(\frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*} \right). \quad (9)$$

Note that when quark phases are changed, the shape of the triangle is invariant, and that if all V_{ij} are real, the triangle reduces to a line. Also, it should be emphasized that the sum of the angles is always $\pi \pmod{2\pi}$ even if the triangle does not close. Thus, $\alpha + \beta + \gamma = \pi$ does not test the unitarity; it simply tests if the angles measured are as defined above. As long as the test of unitarity is concerned, the measurements of the absolute values of the sides of the triangle is just as important as those of the angles.

Experimentally, the unitarity triangle is already over-constrained. Primary inputs are, (1) $|V_{ub}/V_{cb}|$ from the semileptonic decays of B , (2) B^0 - \bar{B}^0 mixing which gives $|V_{td}|$, (3) and ϵ_K . The upper limit on the B_s mixing also contribute, but to a lesser degree than the above three. When the unitarity triangle is normalized to the length of the bottom ($|V_{cd}V_{cb}|$), each of the three measurements above form a band of for the location of the tip of the triangle. Many such fit have been performed and now the consensus is that the three line cross at a single point within errors. This already supports the standard model of CP violation. In one such fit,[1] the value of $\sin 2\beta$ is predicted as

$$\sin 2\beta = 0.698 \pm 0.066. \quad (10)$$

CP violation (CPV) in B decay may be classified into three categories:

1. CPV in the neutral B mixing which manifests as the particle-antiparticle imbalance in the physical neutral B states ($B_{a,b}$); namely, $|\langle B^0 | B_{a,b} \rangle|^2 \neq |\langle \bar{B}^0 | B_{a,b} \rangle|^2$,
2. CPV by the mixing-decay interference which can occur when both B^0 and \bar{B}^0 can decay to the same final state f , and
3. CPV in decay; namely, the asymmetries in instantaneous decay rates: $|Amp(B \rightarrow f)| \neq |Amp(\bar{B} \rightarrow \bar{f})|$ which can happen when there are multiple diagrams with different weak phases and different strong phases.

¹ Another common notation is $(\alpha, \beta, \gamma) \equiv (\phi_2, \phi_1, \phi_3)$.

2 CPV in mixing

Assuming CPT , the eigenstates of mass and decay rate can be written as

$$\begin{cases} B_a &= pB^0 + q\bar{B}^0 & (m_a, \gamma_a) \\ B_b &= pB^0 - q\bar{B}^0 & (m_b, \gamma_b) \end{cases}.$$

The asymmetry in B^0, \bar{B}^0 contents is the same for B_a and B_b and given by

$$\delta \equiv \frac{|\langle B^0 | B_{a,b} \rangle|^2 - |\langle \bar{B}^0 | B_{a,b} \rangle|^2}{|\langle B^0 | B_{a,b} \rangle|^2 + |\langle \bar{B}^0 | B_{a,b} \rangle|^2} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}.$$

The flavor contents may be measured by the lepton sign in the semileptonic decays. Since $\gamma_a \sim \gamma_b$, one cannot separate B_a and B_b by lifetime as in the case of the neutral kaon system. On $\Upsilon(4S)$, however, one can measure the same-sign dilepton asymmetry where both B 's decay semileptonically:[2]

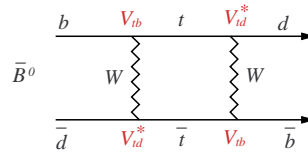
$$A_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} \sim 2\delta.$$

There is also a CP asymmetry in single lepton yield which can be measured whenever equal number of B and \bar{B} are generated.[3] Assuming leptons from neutral and charged B 's cannot be separated,

$$A_\ell \equiv \frac{N_{\Upsilon(4S) \rightarrow \ell^+} - N_{\Upsilon(4S) \rightarrow \ell^-}}{N_{\Upsilon(4S) \rightarrow \ell^+} + N_{\Upsilon(4S) \rightarrow \ell^-}} = \chi \delta, \quad \chi(\text{mixing parameter}) \sim 0.17.$$

This holds even for the quantum-correlated B pair from $\Upsilon(4S)$.[4]

In the standard model, the dominant diagram for mixing is the box diagram and gives


 $\frac{q}{p} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \eta_B, \quad (11)$

where η_B is the (arbitrary) CP phase of B^0 : $CP|B\rangle = \eta_B|\bar{B}\rangle$. We see that q/p is a pure phase; $|p| \neq |q|$ is caused at a higher-order by the interference of the above diagram with the same one with t replaced by c :

$$\delta \sim -2\pi \frac{m_c^2}{m_t^2} \Im \left(\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right) \sim 10^{-3} \text{ (short distance)}.$$

The value of δ , however, is likely to be dominated by long-distance effects such as $D\bar{D}$ intermediate states; it has a large theoretical uncertainty and even the

sign is not reliably predicted.[5] This means that one cannot determine CKM phases from δ . If δ is found at percent level, however, it may signal new physics, and its measurement has an engineering value since δ is assumed to be zero in most calculations.

Experimental results are[6, 7]

$$\delta = \begin{cases} 0.0070 \pm 0.0206 \pm 0.0030 & (CLEO1993) \\ -0.004 \pm 0.014 \pm 0.006 & (OPAL1997) \end{cases},$$

where the OPAL result was actually obtained by fitting the time dependence of tagged semileptonic decays of B 's on Z^0 .

3 CPV by the mixing-decay interference

The flavor-tagged time-dependent decay distribution of the neutral B meson system to a CP eigenstate f ² is given by

$$\Gamma_{B,\bar{B} \rightarrow f}(t) = N e^{-\gamma|t|} \left[1 \pm \Im \left(\frac{q\bar{A}}{pA} \right) \sin \delta m t \right], \quad (12)$$

where N is a normalization factor, $\delta m \equiv m_a - m_b$, $A \equiv \text{Amp}(B^0 \rightarrow f)$, $\bar{A} \equiv \text{Amp}(\bar{B}^0 \rightarrow f)$, and we have assumed $\gamma_a = \gamma_b \equiv \gamma$. This expression applies not only to a pure B^0 or \bar{B}^0 state at $t = 0$, but also to the $\Upsilon(4S)$ system by the replacement $t \rightarrow \Delta t \equiv t_1 - t_2$ where t_1 is the signal-side decay time and t_2 is the tagging-side decay time, and with the understanding that $\Gamma_{B \rightarrow f}$ ($\Gamma_{\bar{B} \rightarrow f}$) applies when the tagging-side was \bar{B}^0 (B^0). This is because, on $\Upsilon(4S)$, if one side decays as B^0 at a proper time t_0 , then the other side is purely \bar{B}^0 at the same proper time t_0 and the evolution after that is the same as that of a single pure \bar{B}^0 prepared at time t_0 . From (12), the time-dependent asymmetry is simply,

$$A_{CP}(t) = \Im \left(\frac{q\bar{A}}{pA} \right) \sin \delta m t. \quad (13)$$

3.1 The gold-plated mode $J/\Psi K_S$

We can estimate $\Im(q\bar{A}/pA)$ for this mode as follows: Since the decay $\bar{B}^0 \rightarrow J/\Psi K_S$ is caused by the quark transition $b \rightarrow c\bar{c}s$, \bar{A} contains the CKM factor $V_{cb}V_{cs}^*$, and since \bar{K}^0 is observed as K_S , \bar{A} should contain $\langle K_S | \bar{K} \rangle$. Similarly, A contains $V_{cb}^*V_{cs}$ and $\langle K_S | K \rangle$. In addition, when a state $|a\rangle$ is related to its CP conjugate state, there appears the CP phase η_a of that state: $CP|a\rangle = \eta_a|\bar{a}\rangle$. In particular,

$$CP|\Psi K^0\rangle = (-)^{L_{\Psi K}} \eta_{\Psi} \eta_K |\Psi \bar{K}^0\rangle,$$

²More precisely, we assumed $|q\bar{A}/pA| = 1$.

where $L_{\Psi K}$ is the orbital angular momentum between Ψ and K . Using the definition $K_S = p_K K^0 - q_K \bar{K}^0$ and the same procedure as (11),

$$\frac{\langle K_S | \bar{K} \rangle}{\langle K_S | K \rangle} = \frac{-q_K^*}{p_K^*} = \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \eta_K^*.$$

Then, the amplitude ratio \bar{A}/A becomes

$$\frac{\bar{A}}{A} = \frac{\langle K_S | \bar{K} \rangle}{\langle K_S | K \rangle} \frac{\langle \Psi \bar{K} | H | \bar{B} \rangle}{\langle \Psi K | H | B \rangle} = \left[\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \eta_K^* \right] \left[(-)^{L_{\Psi K}} \eta_{\Psi} \eta_K \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \eta_B^* \right]$$

Combining this with (11) and noting $\eta_{\Psi} = +1$ (regardless of the CP phase of charm quark) and $L_{\Psi K} = 1$, we get

$$\frac{q\bar{A}}{pA} = \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)^* \bigg/ \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right) \rightarrow \Im \left(\frac{q\bar{A}}{pA} \right) = -\sin 2\beta$$

where we have used the exact definition of the angle β as given by (9). The arbitrary CP phases η_B and η_K are canceled out, and the result is also invariant of the quark phases.

Several experiments have attempted the measurement of $\sin 2\beta$. Here, the analysis by Belle is shown because of the author's familiarity with the experiment. In the c.m. system of $\Upsilon(4S)$, a B meson has a fixed energy and a fixed absolute momentum. If it decays to a set of daughter particles, then

$$E_{\text{tot}} \equiv \sum_{i=1}^n E_i = \frac{m_{\Upsilon(4S)}}{2} = 5.29 \text{ GeV}, \quad P_{\text{tot}} = \left| \sum_{i=1}^n \vec{P}_i \right| = 0.34 \text{ GeV}/c,$$

where (E_i, \vec{P}_i) is the 4-momentum of the i -th daughter. One could thus plot E_{tot} vs P_{tot} to look for a peak at the expected location; historically, however, two equivalent parameters, ΔE and m_{bc} (the 'beam-constrained' mass) are used:

$$\Delta E \equiv E_{\text{tot}} - \frac{m_{\Upsilon(4S)}}{2}, \quad m_{\text{bc}} \equiv \sqrt{\left(\frac{m_{\Upsilon(4S)}}{2} \right)^2 - P_{\text{tot}}^2}.$$

Figure 1 shows the ΔE - m_{bc} plot and its projections for the $B \rightarrow J/\Psi K_S$ candidates corresponding to 10.5 fb^{-1} of data. The analysis also used the modes $\Psi' K_S$, $\chi_{c1} K_S$, $\eta_c K_S$ ($CP = -1$) and $\Psi \pi^0$, ΨK_L ($CP = +1$). The flavor tagging used K^\pm and π^\pm as well as leptons. The asymmetry flips sign for different CP eigenvalues. The resulting value of $\sin 2\beta(\sin 2\phi_1)$ and the time-dependent asymmetry is shown in Figure 2. Figure 1 shows the ΔE - m_{bc} plot and its projections for the $B \rightarrow J/\Psi K_S$ candidates. The measurements of $\sin 2\beta$ are summarized in Table 1. The numbers are consistent with the 'prediction' (10) of the standard model.

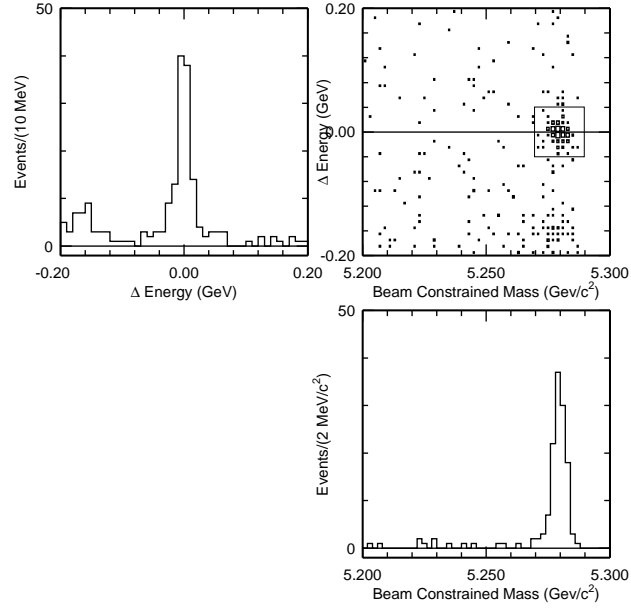


Figure 1: The ΔE - m_{bc} plot and its projections for the $B \rightarrow J/\Psi K_S$ candidates (Belle).

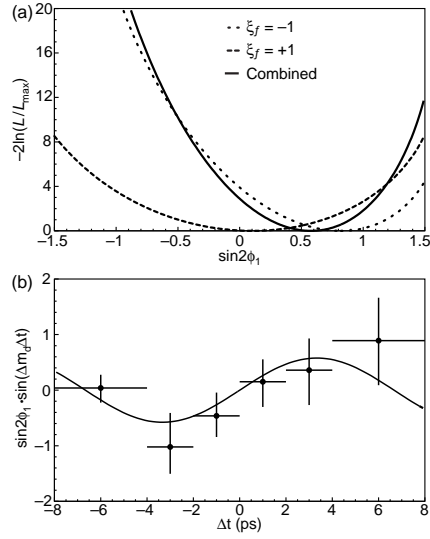


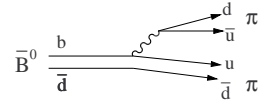
Figure 2: (a) The χ^2 plot for $\sin 2\beta(\sin 2\phi_1)$ for $CP+$ modes, $CP-$ modes, and combined. (b) The time-dependent asymmetry adjusted for CP eigenvalues of the final state. (Belle)

Table 1: Measurements of $\sin 2\beta$.

experiment	$\sin 2\beta$	ref.
OPAL	$3.2^{+1.8}_{-2.0} \pm 0.5$	[8]
ALEPH	$0.84^{+0.82}_{-1.04} \pm 0.16$	[9]
CDF	$0.79^{+0.41}_{-0.44}$	[10]
BaBar	$0.34 \pm 0.20 \pm 0.05$	[11]
Belle	$0.058^{+0.32+0.09}_{-0.34-0.10}$	[12]

3.2 The $\pi^+\pi^-$ final state

The tree diagram for $\bar{B}^0 \rightarrow \pi^+\pi^-$ is caused by the quark-level transition $b \rightarrow u\bar{u}d$, and thus \bar{A}/A is



$$\frac{\bar{A}}{A} = \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\eta_B^*.$$

Together with (11), the asymmetry coefficient for this mode is

$$\Im\left(\frac{q}{p} \cdot \frac{\bar{A}}{A}\right) = \Im\left(-\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\eta_B \cdot \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\eta_B^*\right) \quad (14)$$

$$= \Im\left[-\left(\frac{V_{tb}^*V_{td}}{-V_{ub}V_{ud}^*}\right) \Bigg/ \left(\frac{V_{tb}^*V_{td}}{-V_{ub}V_{ud}^*}\right)^*\right] = -\sin 2\alpha, \quad (15)$$

where the arbitrary CP phase η_B again cancelled out and we have used the definition of the angle α given in (9).

Since the $\pi^+\pi^-$ mode is already observed at ~ 1.5 events/ fb^{-1} , we can expect about 450 events at 300 fb^{-1} where the background would have improved, say, by a better vertexing. This together with the effective tagging efficiency of 0.27, the error on $\sin 2\alpha$ will be about 0.15. There is, however, a complication caused by the $b \rightarrow d$ penguin transition which has a different weak phase from that of the tree transition. Since the isospin-2 component does not receive contribution from the penguin, one may extract it by combining with $B^- \rightarrow \pi^-\pi^0$ and $\bar{B}^0 \rightarrow \pi^0\pi^0$ and applying an isospin analysis.[17] The detection of $\pi^0\pi^0$ mode, however, is experimentally challenging and the method suffers from a reduction of statistical power. A more promising way may be provided by the QCD factorization approach[13] with a systematic heavy-quark expansion which indicates that $\sin 2\alpha$ can be determined with a small theoretical error (of order 0.1) from the asymmetry coefficient $\Im(q\bar{A}/pA)$ albeit with a discrete ambiguity.

3.3 Flavor-specific final states

CPV by mixing-decay interference can occur even if the final state is not a CP eigen state as long as both B^0 and \bar{B}^0 can decay to the same final state. One example is the $D^+\pi^-$ final state[14] where $\bar{B}^0 \rightarrow D^+\pi^-$ caused by $b \rightarrow c\bar{u}d$ and $B^0 \rightarrow D^+\pi^-$ cause by $\bar{b} \rightarrow \bar{u}c\bar{d}$ interfere through mixing. The rate of a pure \bar{B}^0 at $t = 0$ decaying to $D^+\pi^-$ at t is

$$\Gamma_{\bar{B}^0 \rightarrow D^+\pi^-}(t) \propto \frac{e^{-\gamma|t|}}{2} \left[(1 + r^2) + (1 - r^2) \cos \delta mt + 2r \sin(\phi_w + \delta) \sin \delta mt \right]$$

with $r \equiv |A(B^0 \rightarrow D^+\pi^-)/A(\bar{B}^0 \rightarrow D^+\pi^-)| \sim 0.02$, $\phi_w = 2\beta + \gamma$ according to the exact definitions (9)[15] and δ is the strong phase. Starting from $\Gamma_{\bar{B}^0 \rightarrow D^+\pi^-}$ given above, $\Gamma_{B^0 \rightarrow D^+\pi^-}$ is obtained by flipping the signs of $\cos \delta mt$ and $\sin \delta mt$, $\Gamma_{B^0 \rightarrow D^-\pi^+}$ by $\phi_w \rightarrow -\phi_w$, and $\Gamma_{\bar{B}^0 \rightarrow D^-\pi^+}$ by both replacements. On $\Upsilon(4S)$, the only modification needed is again $t \rightarrow \Delta t$.

If we set $\delta = 0$, the CP asymmetry between the two favored modes ($\Gamma_{\bar{B}^0 \rightarrow D^+\pi^-}$ and $\Gamma_{B^0 \rightarrow D^-\pi^+}$) is $\sim 0.01 \sin \phi_w$ and that between the two suppressed modes ($\Gamma_{\bar{B}^0 \rightarrow D^-\pi^+}$ and $\Gamma_{B^0 \rightarrow D^+\pi^-}$) is $\sim 0.06 \sin \phi_w$. The statistics of the favored modes is about 5 times that of the suppressed modes; thus, the CPV information is mostly contained in the suppressed modes. The measurement of these 4 modes give two quantities: $r \sin(\phi_w - \delta)$ and $r \sin(\phi_w + \delta)$. Thus, the value of r needs to be input externally in order to extract $\phi_w = 2\beta + \gamma$.

One could also use $D^{*+}\pi^-$ where D^0 is not reconstructed in the decay $D^{*+} \rightarrow D^0\pi^+$, which enhances the statistics. The expected precision for a given luminosity may be expressed as $\sigma_{\sin(2\beta+\gamma)} = 4 \sim 5\sigma_{\sin 2\beta}$.

A similar mechanism for CPV can be found in the $D^{*+}\rho^-$ mode.[15, 16] The asymmetries again are of order $1 \sim 5\%$; this time, however, there are interferences among the three polarization amplitudes each of which evolves as a function of time. One thus measures the angular correlation of the decays $D^{*+} \rightarrow D^0\pi^+$ and $\rho^- \rightarrow \pi^+\pi^0$ at a given time. The relevant weak phase is the same as that of $D^+\pi^-$: $\phi_w = 2\beta + \gamma$, but there are more degrees of freedom for the measurements. The statistic-enhancing partial reconstruction technique as the one used for $D^{*+}\pi^-$ is probably not realistic due to the requirement to measure the decay angles. The statistical power, however, is expected to be comparable to that of $D^{*+}\pi^-$.

4 CPV in decay

The particle-antiparticle asymmetry in partial decay rate can occur when there are multiple diagrams with different weak phases (i.e. the CKM phases) and different strong phases. Here, we will look at two historically important categories of modes: DK and $K\pi$, $\pi\pi$ modes. There are, however, many other

modes that are just as important in studying CP violation such as $B \rightarrow$ a light pseudoscalar plus a light vector.

4.1 $B \rightarrow DK$

One clean example is $B^- \rightarrow D_{1,2}K^-$ (and its charge conjugate mode) where $D_{1,2} \equiv (D^0 \pm \bar{D}^0)/\sqrt{2}$, D_1 is detected by the final states K^+K^- , $\pi^+\pi^-$, etc. and D_2 by $K_S\pi^0$, $K_S\rho^0$, $K_S\phi$, etc. Then, $A(B^- \rightarrow D_1K^-)$ is the sum of $A(B^- \rightarrow D^0K^-) \equiv ae^{i\phi_c}e^{i\delta_c}$ and $A(B^- \rightarrow \bar{D}^0K^-) \equiv be^{i\phi_u}e^{i\delta_u}$, where $\phi_{c,u}$ are the phases of the CKM factors (‘weak’ phases), $\delta_{c,u}$ are the strong phases, and a, b are positive. For the charge-conjugate modes, the CKM factors are complex-conjugated but the strong phases stay the same. The decay rates of $B^\mp \rightarrow D_1K^\mp$ are then

$$\Gamma(B^\mp \rightarrow D_l K^\mp) = \frac{|a|^2}{2} [1 + r^2 + (-)^l 2r \Re(e^{\pm i\Delta\phi} e^{i\Delta\delta})] \quad (16)$$

where $l = 1, 2$, $r \equiv b/a$, $\Delta\phi \equiv \phi_u - \phi_c$, and $\Delta\delta \equiv \delta_u - \delta_c$. For a given l , we see that there is an particle-antiparticle asymmetry if $\Delta\phi \neq 0$ and $\Delta\delta \neq 0$.

Once r is measured by flavor-specific modes of D^0 , the measurements of the two modes $\Gamma(B^\mp \rightarrow D_1K^\mp)$ (or $l=2$) allows a determination of $\Delta\phi$ and $\Delta\delta$ by a triangle construction.[18] Experimentally, however, it would be simpler to fit simultaneously the all 4 numbers $\Gamma(B^\mp \rightarrow D_l K^\mp)$ ($l = 1, 2$) (each normalized to $\Gamma(B^- \rightarrow D^0 K^-)$). There is an experimental difficulty in measuring r by hadronic final states because of the doubly-cabbibo-suppressed decays[19] which causes interference between $A(B^- \rightarrow D^0 K^-)$ and $A(B^- \rightarrow \bar{D}^0 K^-)$. [19] This, however, can be used to extract r as well as $\Delta\phi$ and $\Delta\delta$ by measuring at least modes, say, $B^- \rightarrow (K^+\pi^-)K^-$ and $B^- \rightarrow (K^-K^+)K^-$ together with their charge-conjugate modes.[19]

The relevant quark diagrams can be better understood by systematically writing down all diagrams for $B^-/\bar{B}^0 \rightarrow DK/D_s\pi$. For B^- , we have

	λ_c	λ_u	
T			
C			$A(B^- \rightarrow D^0 K^-) = \lambda_c(T_c + C_c)$ $A(B^- \rightarrow \bar{D}^0 K^-) = \lambda_u(C_u + A)$ $A(B^- \rightarrow D^- \bar{K}^0) = \lambda_u A$ $A(B^- \rightarrow D_s^- \pi^0) = \frac{1}{\sqrt{2}} \lambda_u T_u$
A			

and those for \bar{B}^0 are

$$\begin{array}{cc}
\begin{array}{c} \lambda_c \\ \text{T} \end{array} & \begin{array}{c} \begin{array}{c} \text{b} \quad \text{c} \quad \text{s} \\ \text{d} \quad \text{d} \quad \text{K}^- \\ \text{d} \quad \text{D}^+ \end{array} \end{array} & \begin{array}{c} \lambda_u \\ \text{T} \end{array} & \begin{array}{c} \begin{array}{c} \text{b} \quad \text{c} \quad \text{s} \\ \text{d} \quad \text{d} \quad \text{D}_s^- \\ \text{d} \quad \text{u} \quad \text{D}^+ \end{array} \end{array} & \begin{array}{l} A(\bar{B}^0 \rightarrow D^+ K^-) = \lambda_c T_c \\ A(\bar{B}^0 \rightarrow D^0 \bar{K}^0) = \lambda_c C_c \\ A(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0) = \lambda_u C_u \\ A(\bar{B}^0 \rightarrow D_s^- \pi^+) = \lambda_u T_u \end{array} \\
\begin{array}{c} \lambda_c \\ \text{C} \end{array} & \begin{array}{c} \begin{array}{c} \text{c} \quad \text{s} \\ \text{d} \quad \text{d} \quad \text{D}^0 \\ \text{d} \quad \text{K}^0 \end{array} \end{array} & \begin{array}{c} \lambda_u \\ \text{C} \end{array} & \begin{array}{c} \begin{array}{c} \text{u} \quad \text{s} \\ \text{d} \quad \text{d} \quad \bar{D}^0 \\ \text{d} \quad \text{K}^0 \end{array} \end{array} & \text{(17)}
\end{array}$$

where $\lambda_c \equiv V_{cb}V_{us}^*$ and $\lambda_u \equiv V_{ub}V_{cs}^*$ are the CKM factors. The strong phases are contained in $T_{c,u}$, $C_{c,u}$, and A , which are the tree, color-suppressed, and annihilation amplitudes, respectively. One notes several interesting features:

1. There is no penguin diagrams. This is because there should be even number of c or \bar{c} quarks in the final state of a penguin diagram and here we have one.
2. There is no annihilation diagram for \bar{B}^0 . Such diagram should have even number of s or \bar{s} quarks in the final state where we have one.
3. We can read off the relations

$$A(D^0 K^-) = A(D^+ K^-) + A(D^0 \bar{K}^0) \quad (18)$$

$$A(\bar{D}^0 K^-) = A(D^- \bar{K}^0) + A(\bar{D}^0 \bar{K}^0) \quad (19)$$

which are nothing but the isospin relations and are valid even with final-state rescatterings such as $B^- \rightarrow D_s^- \pi^0 \rightarrow \bar{D}^0 K^-$ or $D^- \bar{K}^0$. In fact, one can *define* $T_{c,u}$ and $C_{c,u}$ by (17) and A by $A(D^- \bar{K}^0)$. 4. $D^- \bar{K}^0$ is a pure annihilation (including the rescattering). If $A(D^- \bar{K}^0)$ turns out to be zero, one can extract b from other less-suppressed modes.[20]

The measured weak phase is $\Delta\phi = \arg(\lambda_u/\lambda_c) \sim -\gamma$. Strictly speaking, however, what is measured depends on the final state of the D decay

$$\Delta\phi = \begin{cases} -\gamma + \xi & (K^+ K^-) \\ -\gamma - \xi & (\pi^+ \pi^-) \end{cases} \quad \xi \equiv \arg \frac{V_{cd}V_{cs}^*}{-V_{ud}V_{us}^*}, \quad (20)$$

where $\xi \sim \lambda^4 \sim 0.002$ in the standard model ($\lambda \sim 0.22$ is the Cabbibo suppression factor). This difference is caused by the small direct CP violation in the D decays. Statistically, one needs about 300 fb^{-1} or more for a viable measurement, and the suppression of background for the suppressed modes is an experimental challenge.

4.2 $B \rightarrow K\pi, \pi\pi$

Tree-penguin interference could cause sizable rate asymmetries in these modes. Since many of them have already been observed, we may find rate asymmetries sometime soon. The extraction of the angle γ , however, is non-trivial.

factorization in the framework of QCD.[13] The theoretical errors are still not small, but at least the uncertainties can now be estimated systematically.

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